How effective is the effective refractive index?

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Abstract: Photonic crystals are often assigned an effective refractive index defined by their dispersion relation. This index can predict their refractive properties consistent with Snell's law, but might not predict their dynamic properties in accordance with the Fresnel formulae.

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Just as a crystal is a periodic array of atoms or molecules that determines the electronic dispersion relation, a photonic crystal is a periodic array of subwavelength homogeneous materials that determines the photonic dispersion relation. A particular class of photonic crystal, a metamaterial, has attracted the attention of the scientific community. When the wavelength of the field interacting with the structure is much longer than the unit cell a, the metamaterial can be treated as a homogeneous dielectric with macroscopic parameters such as effective refractive index n_{eff}. Proper choice of component materials and geometries can yield metamaterials with novel optical properties that allow them to control light in unconventional ways [1].

While traditional methods of deriving an effective refractive index apply in the long-wavelength limit [2–4], photonic crystals consisting of thin wires can have optical properties of a low-loss dielectric at wavelengths much larger than the wire thickness, but only slightly larger than twice the unit cell [5, 6]. Dielectric photonic crystals have also been assigned an effective refractive index $n^{\text{d}}_{\text{eff}}$ based on their dispersion surface at wavelengths only slightly larger than the unit cells [7–9].

Here, we analyze the limits of applicability of $n^{\text{d}}_{\text{eff}}$ as an effective refractive index. The dispersion relation method derives the index from the dispersion characteristics of an infinite periodic structure leading to $n^{\text{d}}_{\text{eff}} = \frac{c_0 k}{\omega}$. In some instances, the index $n^{\text{d}}_{\text{eff}}$ is independent of the propagation direction so it defines a circular equifrequency surface in k-space for 2D photonic crystals. While dielectric photonic crystals with an effective index $n^{\text{d}}_{\text{eff}}$ have been shown to refract light as a homogeneous dielectric with the same refractive index, it has yet to be demonstrated whether their reflectivity and transmissivity match those of a homogeneous material with the same refractive index.

In our analysis, we introduce two other methods of defining the effective refractive index of a photonic crystal. The first method matches the angle-dependent reflectivity $\mathcal{R}(\theta)$ at the interface of a finite photonic

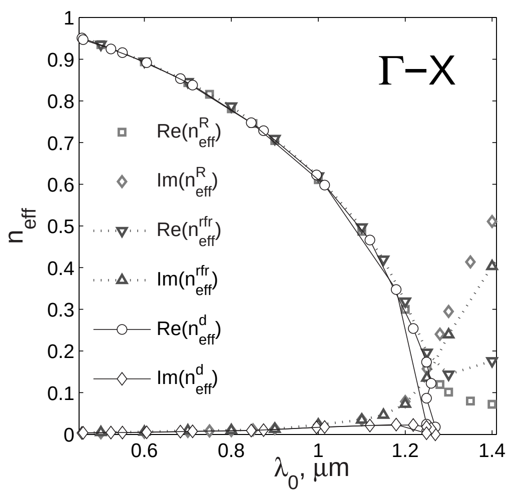

Fig. 1: Refractive index (real and imaginary parts) of the metamaterial as a function of free-space wavelength as predicted by its dispersion curve ($n^{\text{d}}_{\text{eff}}$) normal incidence refraction ($n^{\text{r}}_{\text{eff}}$) and angle dependent reflectivity ($n^{\text{R}}_{\text{eff}}$). Wire radius: $r = 15$ nm, unit cell size: $a = 200$ nm.

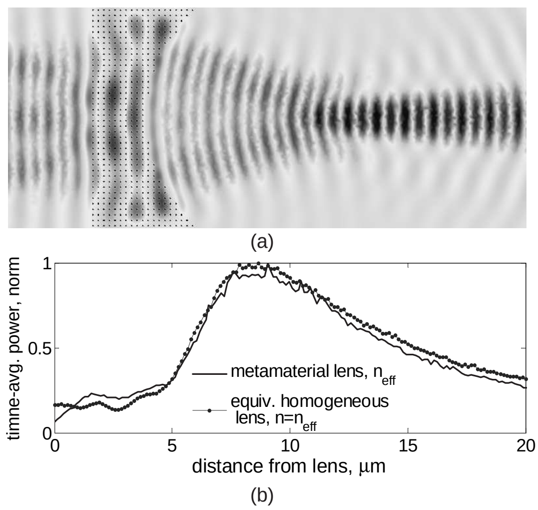

Fig. 2: (a) Finite-element simulation of $\lambda_0 = 1$ μm light incident on a plano-concave ULIM lens ($n_{\text{eff}} = 0.61 + 0.022i$) with a radius of curvature R=5 μm. (b) On-axis power of plane wave focused by ULIM lens and a homogeneous lens with refractive index $n = n_{\text{eff}}$.

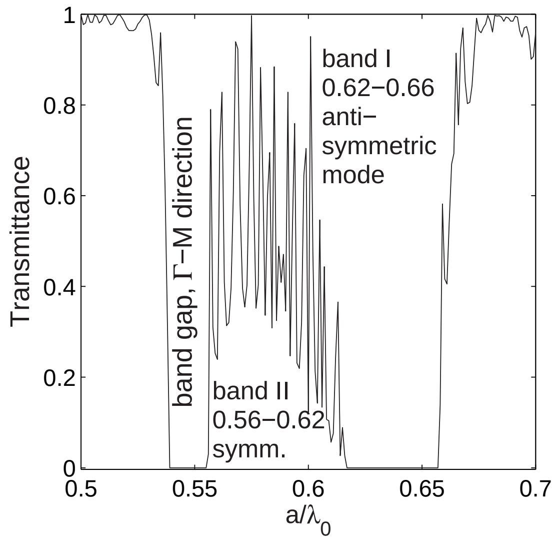

Fig. 3: Transmission (TE) through 64 periods of the hexagonal photonic crystal consisting of GaAs cylinders oriented in the Γ-M direction. Note that symmetric modes correspond to non-zero transmission regions, while anti-symmetric modes correspond to zero-transmission regions (insets). The lines in the mode-field plots denote the Γ-M propagation direction.

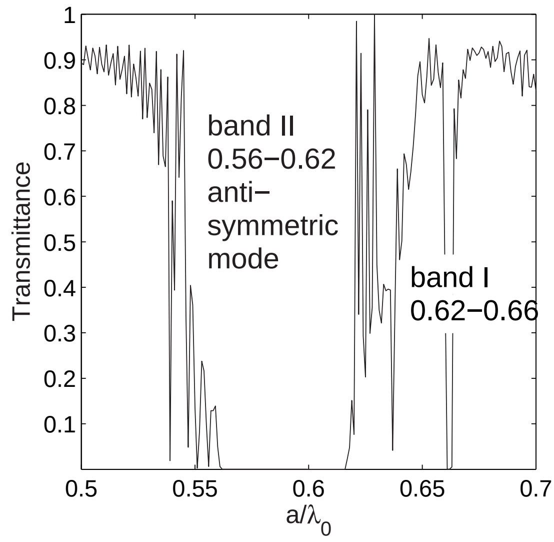

Fig. 4: Transmission (TE) through 64 periods of the hexagonal photonic crystal consisting of GaAs cylinders oriented in the Γ-K direction. Note that symmetric modes correspond to non-zero transmission regions, while anti-symmetric modes correspond to zero-transmission regions (insets). The lines in the mode-field plots denote the Γ-K propagation direction.

crystal structure to that of a homogeneous material, leading to $n_{\text{eff}}^{\text{R}}$. The second considers the normal-incidence refraction into a finite structure to determine the wavelength of the Bloch wave (in the plane-wave limit) inside the photonic crystal. This leads to $n_{\text{eff}}^{\text{r}} = \frac{\lambda_0}{\lambda}$.

We start by applying the various definitions of phase effective index to a metamaterial composed of silver wires embedded in air. Because of the losses of silver at optical frequencies, these metamaterials are inherently lossy and present a complex refractive index. Ref. [5] showed that for TM modes (electric field parallel to wires), these metal-dielectric metamaterials can be tailored to present the real part of the effective index below unity with the imaginary part significantly lower than bulk metal.

We calculated the effective refractive index using the three methods described above. These independent calculations provide consistent results for both the real and imaginary parts of the index in a wide frequency range for the two main directions Γ-X and Γ-M. As shown in Fig. 1, this photonic crystal behaves like an ultralow index metamaterial (ULIM) at visible wavelengths.

An ultralow index metamaterial has some interesting implications. A beam incident on a planar interface between air and a ULIM, for example, would be refracted away from the normal, as opposed to refracting toward the normal as it would with most optical materials. If one were to build a plano-concave lens with a ULIM, the result would be a converging device, as shown in Fig. 2a. Figure 2b shows that both the location and intensity of the beam focused by the metamaterial lens agrees with that of the equivalent lens made of a homogeneous dielectric with refractive index $n = n_{\text{eff}}^{\text{d}}$.

Certain two-dimensional photonic crystals have circular equifrequency surfaces that correspond to an effective refractive index $\left|n_{\text{eff}}^{\text{d}}\right| < 1$ [7, 8]. In what follows, we study reflection and transmission properties of two of these photonic crystals.

The first photonic crystal is an hexagonal array of GaAs cylinders($n_{\text{GaAs}} = 3.6$) with diameter $2r = 0.7a$, where a is the unit cell length. For TE modes (magnetic field parallel to wires), this structure has a circular equifrequency surface for $\omega = \frac{\nu a}{c_0}$ between 0.59 and 0.62, with corresponding $n_{\text{eff}}^{\text{d}}$ between -0.8 and 0.5. Following Ref. [7], band I corresponds to the frequencies with a positive effective index, and band II corresponds to those with a negative index.

Modes of a photonic crystal are either symmetric or antisymmetric about their symmetry axis. Consider a plane wave incident on a finite photonic crystal such that $\mathbf{k}$ is parallel to the photonic crystal's symmetry axis. Since the incident wave is symmetric about this axis, it will excite only a symmetric mode [10]. If the only mode at this frequency is antisymmetric, all light will be reflected.

Inspection of the mode symmetries reveals that for band I, the Γ-M mode is antisymmetric, and for band II, the Γ-K mode is antisymmetric. For these modes, the reflection coefficient is unity, which contradicts the Fresnel equations predictions given a non-zero $n_{\text{eff}}^{\text{d}}$ for the photonic crystal (see Figs. 3 and 4).

Lastly, we consider the photonic crystal geometry proposed by Gralak *et al* [8], of a square array of dielectric cylinders (n=3) in air. For TM mode, this structure has a circular equifrequency surface in a small

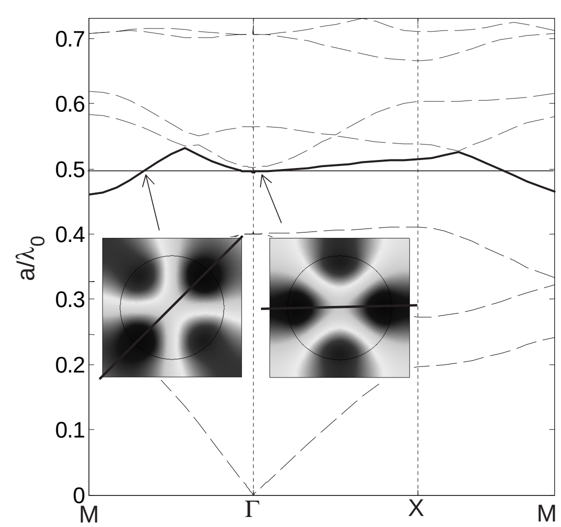

Fig. 5: Dispersion diagram (TM) of a two-dimensional square photonic crystal, unit cell a, consisting of dielectric cylinders of radius $r = 0.374a$ and refractive index $n = 3.0$. The insets show the electric field of modes with frequency $\frac{a}{\lambda_0}$=0.496.

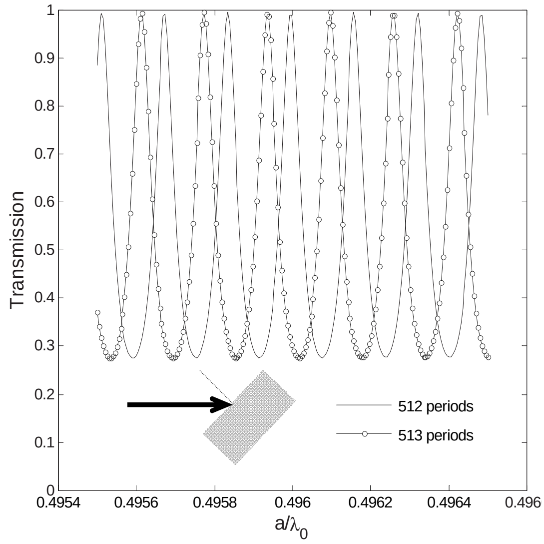

Fig. 6: Transmission through square photonic crystal slab for light incident 45° to the interface. Since the transmission minima are both greater than zero and depend on the length of the photonic crystal slab, none of them are due to total external reflection occurring at a specific frequency.

frequency range near $\frac{a}{\lambda_0} = 0.496$, just above the band gap (see Fig. 5), with corresponding $n^{\mathrm{d}}_{\mathrm{eff}} \approx 0.05$. Unlike the hexagonal photonic crystal discussed above, both of the Γ-X and Γ-X modes are symmetric at $\frac{a}{\lambda_0} = 0.496$, and hence can be excited by an incident plane wave.

If $n^{\mathrm{d}}_{\mathrm{eff}} \approx 0.05$ in this frequency range, a finite photonic crystal slab should not transmit light incident upon it at angles exceeding the critical angle, which is less than 3°. Yet, this is not the case, as shown in Fig. 6, which shows the transmission of light incident at 45° to the normal. This demonstrates that $n^{\mathrm{d}}_{\mathrm{eff}}$ does not necessarily predict a photonic crystal's reflectivity.

While $n^{\mathrm{d}}_{\mathrm{eff}}$ predicts reflection and transmission properties in accordance with the Fresnel formulae for the thin-silver-wire photonic crystal, it does not for the dielectric photonic crystals considered here. One necessary condition for the Fresnel formulae to hold is that the mode in question be symmetric, so an incident plane wave can couple into the finite photonic crystal. A sufficient condition is that the photonic crystal mode must resemble a plane wave, as these are assumed in the derivation of the Fresnel formulae. This occurs in the long wavelength regime and when the inclusions in a metamaterial are much smaller than the wavelength (even if the unit cell is not).

In conclusion, an effective refractive index of a photonic crystal defined by its dispersion surface does not necessarily predict its reflection and transmission properties according to the Fresnel formulae, and hence a photonic crystal can not be considered as equivalent to a homogeneous material with the that refractive index.

References

1. "Focus Issue: Negative Refraction and Metamaterials," Optics Express **11,** 639–838 (2003).
2. D. E. Aspnes, "Local-field effects and effective-medium theory: A microscopic perspective," Am J. Phys. **50,** 704–709 (1982).
3. P. Lalanne and D. Lemercier-Lalanne, "On the effective medium theory of subwavelength periodic structures," J. Mod. Opt. **43,** 2063–2085 (1996).
4. P. Halevi, A. A. Krokhin, and J. Arriaga, "Photonic crystal optics and homogenization of 2D periodic composites," Phys. Rev. Lett. **82,** 719–722 (1999).
5. B. T. Schwartz and R. Piestun, "Total external reflection from metamaterials with ultralow refractive index," J. Opt. Soc. Am. B **20,** 2448–2453 (2003).
6. B. T. Schwartz and R. Piestun, "Waveguiding in air by total external reflection from ultra-low index metamaterials," App. Phys. Lett. **85,** 1–3 (2004).
7. M. Notomi, "Theory of light propagation in strongly modulated photonic crystals: Refraction-like behavior in the vicinity of the photonic bandgap," Phys. Rev. B **62,** 10696–10705 (2000).
8. B. Gralak, S. Enoch, and G. Tayeb, "Anomalous refractive properties of photonic crystals," J. Opt. Soc. Am. A **17,** 1012–120 (2000).
9. A. Berrier, M. Mulot, M. Swillo, M. Qiu, L. Thyln, A. Talneau, and S. Anand, "Negative refraction at infrared wavelengths in a two-dimensional photonic crystal," Phys. Rev. Lett. **93,** 073902(4) (2004).
10. K. Sakoda, "Symmetry, degeneracy, and uncoupled modes in two-dimensional photonic lattices," Phys. Rev. B **52,** 7982–7986 (1995).