APPLIED PHYSICS LETTERS VOLUME 85, NUMBER 1 5 JULY 2004

Waveguiding in air by total external reflection from ultralow index metamaterials

Brian T. Schwartz[a] and Rafael Piestun
Department of Electrical and Computer Engineering, University of Colorado at Boulder, Boulder, Colorado 80309-0425

(Received 14 November 2003; accepted 27 April 2004)

Metamaterials composed of metal-dielectric nanostructures can be engineered to have the real part of the effective refractive index less than unity at optical wavelengths. These materials show intriguing optical properties including total external reflection. We utilize this effect to design and analyze slab waveguide structures that guide visible light in an air core. © *2004 American Institute of Physics*. [DOI: 10.1063/1.1764596]

Optical micro- and nanostructured materials are intriguing because of their ability to control light in unconventional ways. Among these are photonic crystals and metamaterials, which have potential applications in integrated photonics.

Metamaterials are an extension of the concept of artificial dielectrics, which were already designed in the 1940s for microwave frequencies.[3] They typically consist of periodic structures of a guest material embedded in a host material. Metamaterials derive their properties from the subwavelength structure of its component materials in a similar way as homogeneous dielectrics derive their optical properties from the nanometer-scale structure of their atoms. When the wavelength of the field interacting with the structure is much longer than the unit-cell dimensions, the metamaterial can be treated as a homogeneous dielectric with macroscopic parameters such as electric permittivity, magnetic permeability, and an effective refractive index n_{eff}.

Lately, several groups have reported physical properties of metamaterials presenting simultaneous negative permittivity and permeability.[2,4] Waveguides composed of these metamaterials have been studied in the microwave region.[5,6] In these and similar studies,[7,8] the metamaterial fills the waveguide core and is assumed to be lossless and homogeneous.

In this letter, by contrast, we focus on the use of metamaterials with $0 \leqslant \mathrm{Re}(n_{\mathrm{eff}}) \leqslant 1$ for the cladding of an air-core waveguide operating at visible wavelengths. We also account for the effects of losses, dispersion, and the metamaterial's inherent inhomogeneities.

In a previous paper, we proposed the use of ultralow refractive-index metamaterials (ULIM) with the real part of the effective index less than unity as a building block for photonic applications.[9] The demonstrated metamaterial was a two-dimensional square array of cylindrical silver wires embedded in an air host medium. The designed ULIM behaved on refraction, reflection, and transmission as a low-loss dielectric with $0 \leqslant \mathrm{Re}(n_{\mathrm{eff}}) \leqslant 1$ for wavelengths between 0.45 μm and 1.2 μm for light polarized parallel to the wires (see Fig. 1). We determined the effective refractive index from refraction and reflection calculations using Fresnel formulae and discussed the effect of total external reflection (TER) from finite slabs of the metamaterial using numerical models. Here, we study the possibility of guiding light in an air core by the effect of TER from ULIMs.

TER occurs when light propagating in vacuum is incident on a medium with a refractive index less than unity at an angle exceeding the critical angle defined by Snell's Law. This effect is very well known for x rays incident on planar interfaces at grazing angles. In the x-ray regime, the index of refraction is expressed in the form $n = 1 - \delta + i\kappa$ where, typically, $\delta \approx 10^{-5}$ and $\kappa \approx 10^{-7}$. For angles exceeding the critical angle, as defined for the lossless case, the refracted waves are inhomogeneous, the refracted angle is very close to $\pi/2$ and the reflectivity is very close to unity.[10] Metamaterials, in contrast, can be engineered to exhibit TER at visible wavelengths with the real part of the refractive index well below unity ($0 < \delta \leqslant 1$) and a very low loss component as compared to bulk metals ($\kappa \ll 1$).

Waveguide design with ULIMs benefits from their additional degrees of freedom available for index specification. In particular, one advantage of using ULIMs is the possibility of designing a waveguide with a lossless and dispersionless core such as air or vacuum. Nevertheless, such a waveguide will still have a lossy and dispersive cladding. In what follows, we show how to design a single mode slab waveguide with ULIM cladding and minimum loss at wave length $\lambda_0 = 0.5$ μm.

We used the transfer-matrix method[9,11,12] to calculate the effective refractive index $n_{\mathrm{eff}}(\lambda_0)$ of metamaterials of different unit cells as a function of silver fill factor. In this letter we use wires with square cross sections instead of cylindrical wires, as square wires allow for more efficient numerical modeling. Figure 2 shows the effective refractive index as a function of wire width b for a square array with unit cell $a = 200$ nm.

For each of these n_{eff} values, we used both analytical[13] and numerical[14] models to calculate the complex propagation constant β for a single-mode slab waveguide with a core index $n_{\mathrm{co}} = 1$ and homogeneous cladding of $n_{\mathrm{cl}} = n_{\mathrm{eff}}$. To ensure low-loss single-mode operation, the core width of each waveguide was 95% of the single-mode cutoff, $d_{co} = \lambda_0 / 2NA$, where $NA = \sqrt{n_{\mathrm{co}}^2 - \mathrm{Re}(n_{\mathrm{cl}}^2)}$ is the numerical aperture.

Figure 3 shows the $\frac{1}{e}$ power attenuation distance $z_c = 1/2\mathrm{Im}(\beta)$ corresponding to the n_{eff} values in Fig. 2. The dotted curves show the loss of waveguides with homogeneous claddings calculated both analytically and numerically.

---

[a] Electronic mail: brian.schwartz@colorado.edu

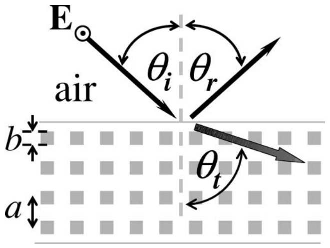

FIG. 1. Refraction and reflection at the interface of air and a two-dimensional metamaterial with ultralow effective refractive index. Light is refracted off the normal to the surface and the refracted waves are inhomogeneous. In this letter the metamaterial is a square array, with period *a*, of silver wires of width *b* and the incident light is polarized parallel to the wires.

To find the loss of the ULIM waveguide (the solid curve in Fig. 3), we replaced the hypothetical homogeneous $0 \leqslant \mathrm{Re}(n_{\mathrm{eff}}) \leqslant 1$ dielectric claddings with their corresponding silver-air ULIMs. The excitation electric field for the numerically modeled waveguides was the single-mode field profile calculated for the corresponding analytical model. We determined the loss using a finite-element Maxwell's equations solver[14] to compute the complex propagation constant β that best fits the electric field along the core center. Computing propagation distance z_c required fitting the electric field in the waveguide to an exponentially decaying field. We achieved convergence to $\pm 3\lambda_0$ using a waveguide of length $9z_c$ and cladding width of nine periods to contain the most weakly guided modes. Note the good agreement between the full finite-element calculations and those using effective index values.

While a waveguide with the thinnest wires minimizes loss,[15] it maximizes fabrication complexity, which is driven by wire cross section and cladding thickness (number of periods required to guide the wave). As shown in Fig. 2, as the wire width decreases, n_{eff} approaches unity ($n_{\mathrm{eff}} \rightarrow 1$). Therefore, the less lossy waveguide modes are weakly guided and more periods of the metamaterial are required to contain them without radiation losses.

Among the metamaterial-cladding geometries considered in Fig. 3, we investigated the performance of a waveguide with metamaterial cladding corresponding to $b/a = 0.2$ ($n_{\mathrm{eff}} = 0.87 + 0.0067i$) as shown in Fig. 4.

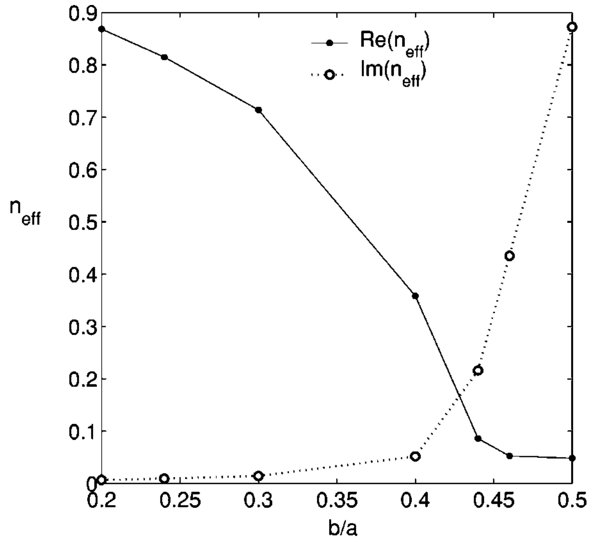

FIG. 2. Refractive index (real and imaginary parts) of the metamaterial as a function of wire width *b* as predicted by its angle-dependent reflectivity. Unit-cell size: $a = 200$ nm, $\lambda_0 = 0.5\ \mu$m.

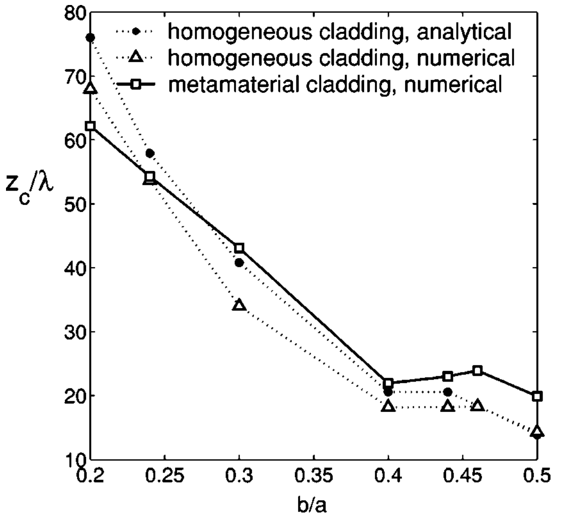

FIG. 3. Power attenuation $\left(\frac{1}{e}\right)$ at $\lambda_0 = 0.5\ \mu$m in single-mode slab waveguides with a homogeneous ultralow index cladding corresponding to the predicted n_{eff} in Fig. 2 and with metamaterial cladding (Unit-cell size: $a = 200$ nm, wire width *b*).

The metamaterial waveguide has a similar loss characteristics and mode profile as the equivalent waveguide with an hypothetical homogeneous $\mathrm{Re}(n_{\mathrm{eff}}) < 1$ cladding (Fig. 4). The mode profile depends only slightly on its location relative to the silver wires. Therefore, one can obtain a first

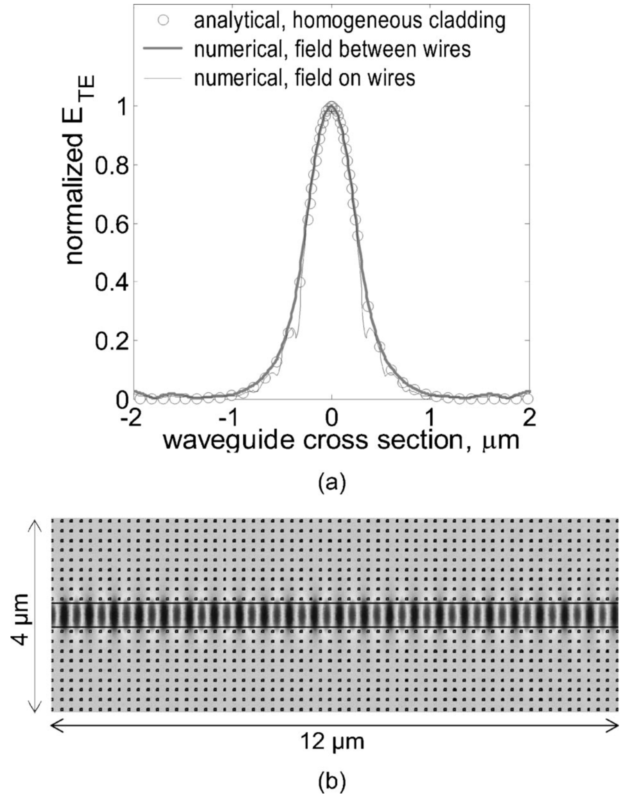

FIG. 4. (a) Comparison of mode profiles ($\lambda_0 = 0.5\ \mu$m) in slab waveguide with core width $d_{\mathrm{co}} = 0.48\ \mu$m. The metamaterial cladding is a square array of $b = 40$ nm square silver wires in a $a = 200$ nm unit cell while the homogeneous cladding has $\mathrm{Re}(n_{\mathrm{cl}}) < 1$ corresponding to the metamaterial's effective refractive index at λ_0 (b) Electric field (TE) of the metamaterial-cladding waveguide mode.

design using the effective index values without need to model the full nanostructured metal-dielectric waveguide. This represents a significant advantage in terms of computation and modeling time.

It is worth comparing the performance of ULIM waveguides to that of other types of waveguides, such as metal waveguides, standard dielectric waveguides, and photonic crystal waveguides.

We first compare the attenuation of the ULIM waveguide with a hollow-metallic waveguide. Silver is best for an air-core slab waveguide with metal cladding because it has relatively low losses at visible wavelengths. At $\lambda_0 = 500$ nm, its refractive index is $n_{\mathrm{Ag}} = 0.13 \pm 2.9i$, which corresponds to a maximum single-mode core width of $d_{\mathrm{co}} = 250$ nm. The TE mode in a silver waveguide with $d = 0.95 d_{\mathrm{co}}$ is attenuated by $\frac{1}{e}$ before it propagates eight wavelengths. As shown in Fig. 3, the $\frac{1}{e}$ power attenuation for the metamaterial waveguide is more than seven times lower than an optimized waveguide with homogeneous silver cladding. This shows that although there is more penetration into the cladding for the ULIM waveguide than for the silver waveguide, the result is still a less lossy design. Clearly, a tradeoff exists between the confinement and the attenuation: For a given unit-cell size, thinner wires lead to higher penetration and lower loss[15] while thicker wires, yield more confinement and higher losses. However, this change is not monotonic as shown in Fig. 3 which shows a local optimum at $b/a \simeq 0.46$.

Silver has also been used to make nanoparticle plasmon waveguides that propagate electromagnetic energy as surface plasmon polaritons.[16] Because of their subwavelength scale, these devices have potential applications in near-field optical microscopy and integrated optical circuits. They can propagate visible light on the order of a wavelength,[17] which is appropriate to these applications. In contrast, the metamaterial waveguide designs presented here do not have the same confinement characteristics but can propagate visible light more than 60 wavelengths.

Photonic crystal waveguides also employ periodic structures with line defects to propagate light by photonic band gap effects, i.e., light in the band gap frequencies cannot propagate into the photonic crystal cladding. These waveguides have promising characteristics for photonic integration but still require tight tolerances to reduce scattering losses.[18,19] Note that ULIM waveguides guide light not by photonic band gap effect but by TER. Therefore, the metamaterial does not require a band gap as shown in Ref. 9.

Since the ULIM waveguide guides light by TER, we expect that tolerances in the metamaterial's periodicity will only slightly change its index producing low additional losses. For a given radius of curvature, bend loss decreases exponentially with the waveguide's profile height parameter $\Delta = (n_{\mathrm{co}}^2 - n_{\mathrm{cl}}^2) / 2\mathrm{n}_{\mathrm{co}}^2$.[20] Therefore, to obtain sharp bends with ULIM waveguides one would need to maximize the index contrast. This fact creates another tradeoff between attenuation and bending diameter. Therefore, tapered designs where the confinement is increased as the waveguide approaches a bend can be envisioned and are currently under investigation.

Note that ULIM waveguides can be designed using conventional methods and tabulated values of the effective index. Rigorous electromagnetic methods are needed only in the final stage of optimization. However, as opposed to conventional waveguides, ULIM waveguides can have an air core.

In conclusion, we have demonstrated the possibility of guiding light in air by TER using metamaterials with low refractive index. The design of air-core metamaterial-cladding waveguides involves the consideration of several tradeoffs. The examples presented show that the flexibility in the specification of the effective properties of a metamaterial opens up opportunities for device applications.

The authors would like to thank Alan R. Mickelson and Dejan Filipovic for their valuable suggestions and insights. This work was partially supported by the National Science Foundation under Grant Nos. IGERT 9870665 and NIRT 0304650.

[1] J. D. Joannopoulos, R. D. Meade, and J. N. Winn, *Photonic Crystals* (Princeton University Press, Princeton, NJ, 1995).
[2] J. Pendry, Opt. Express **11**, 639 (2003).
[3] J. Brown, Prog. Dielectr. **2**, 193 (1960).
[4] R. A. Shelby, D. R. Smith, and S. Schultz, Science **292**, 77 (2001).
[5] I. S. Nefedov and S. A. Tretyakov, e-print in arXiv: cond-mat/0211185 v1 (2002), at http://arxiv.org/pdf/cond-mat/0211185.
[6] C. Caloz, C. C. Chang, and T. Itoh, J. Appl. Phys. **90**, 5483 (2001).
[7] K. Halterman, J. M. Elson, and P. L. Overfelt, Opt. Express **11**, 521 (2003).
[8] A. C. Peacock and N. G. R. Broderick, Opt. Express **11**, 2502 (2003).
[9] B. T. Schwartz and R. Piestun, J. Opt. Soc. Am. B **20**, 2448 (2003).
[10] D. Attwood, *Soft X-Rays and Extreme Ultraviolet Radiation: Principles and Applications* (Cambridge University Press, Cambridge, MA, 1999).
[11] P. M. Bell, J. B. Pendry, L. M. Moreno, and A. J. Ward, Comput. Phys. Commun. **85**, 306 (1995).
[12] A. L. Reynolds, *Translight Software*, Optoelectronics Research Group, Dept. Electronics and Electrical Engineering (University of Glasgow, UK, 2000).
[13] A. Reisinger, Appl. Opt. **12**, 1015 (1973).
[14] Femlab, Comsol Corporation Burlington, MA (version 2.30.153).
[15] This is true so long as the silver wires are thick enough to retain the optical properites of bulk silver. The refractive index of silver films thinner than $d \approx 15$ nm have been shown to diverge from their bulk values. While a similar effect might occur for wires of this width, the smallest wires considered in this paper are 40-nm thick. See I. An and K. Oh, J. Kor. Opt. Soc. **29**, 370 (1996); U. Kreibig and M. Vollmer, *Optical Properties of Metal Clusters* (Springer, Berlin, 1995).
[16] M. Quinten, A. Leitner, J. R. Krenn, and F. R. Aussenegg, Opt. Lett. **23**, 1331 (1998).
[17] S. A. Maier, P. O. Kik, H. A. Atwater, S. Meltzer, E. Harel, B. Koel, and A. A. G. Requicha, Nat. Mater. **2**, 229 (2003).
[18] A. Cucinotta, S. Selleri, L. Vincetti, and M. Zoboli, Appl. Phys. B: Lasers Opt. **73**, 595 (2001).
[19] A. Cucinotta, S. Selleri, L. Vincetti, and M. Zoboli, J. Lightwave Technol. **20**, 1433 (2002).
[20] A. W. Snyder and J. D. Love, *Optical Waveguide Theory*, 2nd ed (Chapman and Hall, 1983).